MATHEMATICS

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AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE
(MAIN + ADVANCE) & COMPETITIVE EXAM.
FOR XII (PQRS)

DIFFERENTIALS, ERRORS AND APPROXIMATIONS
& Their Properties

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THINGS TO REMEMBER

1. Let y = f(x) be a function of x, and let Δx be a small change in x and Δy be the corresponding change in y. Then,

    $$\Delta y = \frac{dy}{dx} \, \Delta x \text{ approximately.}$$

    $\frac{dy}{dx} \, \Delta x$ is called differential of y and is denoted by dy.

2. Following are some useful results on differentials :

    (i) If f(x) is a constant function, then its differential is zero.

    (ii) If y = cu, then dy = c du, c is a constant.

    (iii)If y = u ± v, then dy = du ± dv

    (iv)If y = iv, then dy = u dv + v du

    (v) If $y = \dfrac{u}{v}$, then $dy = \dfrac{v\,du - u\,dv}{v^2}$

    (vi)If y = f(x), then dy = f′(x) dx.

3. (i) Let y = f(x) be a given function of x. If Δx is an error in x, then the corresponding error Δy in y is given by

    $$\Delta y = \frac{dy}{dx} \, \Delta x$$

    (ii) If Δx is an error in x, then $\dfrac{\Delta x}{x}$ is called relative error in x.

    (iii)If Δx is an error in x, then $\dfrac{\Delta x}{x} \times 100$ is called the percentage error in x.

EXERCISE-1

1. If y = x⁴ − 10 and if x changes from 2 to 1.99, what is the approximate change in y ?

2. A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate if the radius of the plate before heating is 10 cm.

3. Find the percentage error in calculating the volume of a cubical box if an error of 1% is made in measuring the length of edges of the cube.

4. The time T of a complete oscillation of a simple pendulum of length *l* is given by the equation.

    $$T = 2\pi\sqrt{\frac{l}{g}},$$

    where g is constant. What is the percentage error in T when *l* is increased by 1% ?

5. Using differentials find the approximate value of tan 46°, if it is being given that 1° = 0.01745 radius.

6. If in a triangle ABC, the side c and the angle C remain constant, while the remaining elements are changed slightly, using differentials show that

$$\frac{da}{\cos A} + \frac{db}{\cos B} = 0$$

7. If a triangle ABC, inscribed in a fixed circle, be slightly varied in such away as to have its vertices always on the circle, then show that

$$\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0$$

8. Find the approximate value of f(3.02), where f(x) $3x^2 + 5x + 3$.

9. Find the approximate value of f(2.01), where f(x) = $4x^2 + 5x + 2$.

10. If the radius of a sphere is measured as 9 cm with an error of 0.03 m, find the approximate error in calculating its surface area.

11. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

12. Find the approximate change in the value V of a cube of side x metres caused by increasing the side by 1%.